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A Global Asymptotic Stable Quasi Variable PID Regulator for Robot Manipulators*

Sifuentes Mijares, Juan[1] and Meza Medina, José Luis[1]

***Abstract*— In this paper, we describe the stability analysis of a nonlinear PID regulator with its gain integral constant and proportional and derivative gains variable. In the integral part has been used a saturation function with the position error. This regulator is analyzed only for robot manipulator with revolute joints and is mathematically demonstrated a global stability of the equilibrium of the system in closed-loop. A Lyapunov function candidate is proposed here and its enough to prove global stability for the equilibrium of the system in closed loop. Some simulation are carried out to illustrate the stability results.**

I. INTRODUCTION

In industrial practice, robots manipulators are usually controlled with traditional PID regulators. The PID regulator is still widely used in industrial applications due to their design simplicity, and its excellent performance, especially in applications in which do not require any component of robot dynamics into his control law [1] [2] [3] [4]. A simple linear and decoupled PID feedback controller with appropriate control gain achieves desired position without any steady-state error. This is the main reason why PID controllers are still used in industrial robots [1], [5], [6].

Normally, the selection of the gains for a PID regulator is considering for constants gains. This characteristic establishes a limit for the application of this controller. In the cases where a asymptotic stability and performance is important to be maintained is necessary selecting variable gain in order to maintain both [7]. Gain scheduling, fuzzy control and neural networks are some of techniques that have been proposed to choose adequate gains depending on the different application for the robot control configuration to reach this. [8], [9], [10], [11], [12], [13].

Although the controller PID for robot manipulators has been widely used in industrial robots, there still exist open problems worthy to be studied. Some of these open problems are the lack of proof of global asymptotic stability [14].

Recently, the stability for a linear PID controller in closed loop with a robot manipulator has been guarantees only in local asymptotic sense [1] [2] [3] [4] [15], or in the best of the cases, in a semiglobal sense [16], [14].

Inspired in the previous works of PID regulators [17] [18], our principal contribution is an analysis and probe of a global stability of the equilibrium in closed loop for a class of regulator nonlinear PID for robots manipulators with only revolute joints. A Lyapunov candidate function is suggested and used with the Lyapunov direct method to show global stability of the system. Our first contribution is the proposal of a new PID regulator, with proportional and derivative variable gains and integral constant gain. Which improbe the previous work in [7] in the fact of not using the knowledge of gravity vector.

Throughout this paper, the vectors are denoted by bold small letters. The norm of vector $\boldsymbol{x}$ is defined as $\|\boldsymbol{x}\| = \sqrt{\boldsymbol{x}^T\boldsymbol{x}}$ and that of matrix $A(\boldsymbol{x})$ is defined as the corresponding induced norm $\|A(\boldsymbol{x})\| = \sqrt{\lambda_{\rm M}\{A(\boldsymbol{x})^T A(\boldsymbol{x})\}}$. We use the notation $\lambda_{\rm m}\{A(\boldsymbol{x})\}$ and $\lambda_{\rm M}\{A(\boldsymbol{x})\}$ to indicate the smallest and largest eigenvalues, respectively, of a symmetric positive definite bounded matrix $A(\boldsymbol{x})$, for any $\boldsymbol{x} \in \mathbb{R}^n$. By an abuse of notation, we define $\lambda_{\rm M}\{A\}$ as the least upper bound (supremum) of $\lambda_{\rm M}\{A(\boldsymbol{x})\}$, for all $\boldsymbol{x} \in \mathbb{R}^n$, that is, $\lambda_{\rm M}\{A\} := \sup_{\boldsymbol{x}\in\mathbb{R}^n} \lambda_{\rm M}\{A(\boldsymbol{x})\}$. Similarly, we define $\lambda_{\rm m}\{A\}$ as the greatest lower bound (infimum) of $\lambda_{\rm m}\{A(\boldsymbol{x})\}$, for all $\boldsymbol{x} \in \mathbb{R}^n$, that is, $\lambda_{\rm m}\{A\} := \inf_{\boldsymbol{x}\in\mathbb{R}^n} \lambda_{\rm m}\{A(\boldsymbol{x})\}$.

The remaining part of the paper is organized as follows. Section 2 presents the robot dynamic model and some important properties. The PID with nonlinear gains and the control law is given in Section 3. A Global asymptotic stability analysis is presented in Section 4. Section 5 remarks some conclusions.

II. ROBOT DYNAMICS

Consider the general equation describing the dynamics of a n-degrees of freedom rigid robot manipulator [19]:

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} \tag{1}$$

where $\boldsymbol{q}$ is the $n \times 1$ vector of joint displacements, $\dot{\boldsymbol{q}}$ is the $n \times 1$ vector of joint velocities, $\boldsymbol{\tau}$ is the $n \times 1$ vector of applied torques, $M(\boldsymbol{q})$ is the $n \times n$ symmetric positive definite manipulator inertia matrix, $C(\boldsymbol{q},\dot{\boldsymbol{q}})$ is the $n \times n$ matrix of centripetal and Coriolis torques, and $\boldsymbol{g}(\boldsymbol{q})$ is the $n \times 1$ vector of gravitational torques obtained as the gradient of the robot potential energy $\mathcal{U}(\boldsymbol{q})$, i.e.:

$$\boldsymbol{g}(\boldsymbol{q}) = \frac{\partial \mathcal{U}(\boldsymbol{q})}{\partial \boldsymbol{q}} \tag{2}$$

We assume that all the links are joined together by revolute joints. Four important properties of dynamics (1) are the following:

Property 1. [20] The matrix $C(\boldsymbol{q},\dot{\boldsymbol{q}})$ and the time derivative $\dot{M}(\boldsymbol{q})$ of the inertia matrix satisfy:

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[1]División de Estudios de Posgrado e Investigación del Instituto Tecnológico de la Laguna. Blvd. Revolución y Czda Cuauhtemoc S/N, Torreón, Coahuila, México, CP. 27000. email: juansm@faraday.itlalaguna.edu.mx jlmeza@faraday.itlalaguna.edu.mx

$$\dot{\boldsymbol{q}}^T \left[\frac{1}{2}\dot{M}(\boldsymbol{q}) - C(\boldsymbol{q},\dot{\boldsymbol{q}})\right]\dot{\boldsymbol{q}} = \boldsymbol{0} \qquad \forall\ \boldsymbol{q},\dot{\boldsymbol{q}} \in \mathbb{R}^n$$

$$\dot{M}(\boldsymbol{q}) = C(\boldsymbol{q},\dot{\boldsymbol{q}}) + C(\boldsymbol{q},\dot{\boldsymbol{q}})^T.$$

Property 2. [21] For robots having only revolute joints, the vector $\boldsymbol{g}\left(\boldsymbol{q}\right)$ is Lipschitz, that is there exist a constant $K_g > 0$ such that

$$\left\|\boldsymbol{g}\left(\boldsymbol{x}\right) - \boldsymbol{g}\left(\boldsymbol{y}\right)\right\| \leq K_g \left\|\boldsymbol{x} - \boldsymbol{y}\right\|$$

Property 3. [22] There exists a positive constant k_c such that for all $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{z} \in \mathbb{R}^n$ we have:

$$\|C(\boldsymbol{x},\boldsymbol{y})\boldsymbol{z}\| \leq k_c\|\boldsymbol{y}\|\ \|\boldsymbol{z}\|$$

If the matrices of the dynamics are known, then the constants k_g and k_c can be obtained using the following expressions [23]:

$$k_g = n\left(\max_{i,j,q}\left|\frac{\partial g_i(\boldsymbol{q})}{\partial q_j}\right|\right), k_c = n^2\left(\max_{i,j,k,q}\left|C_{k_{ij}}(\boldsymbol{q})\right|\right)$$

where $C_{k_{ij}}(\boldsymbol{q})$ is a matrix whose element $\{i,j\}$ is the c_{ijk} Christoffel symbol of $C(\boldsymbol{q},\dot{\boldsymbol{q}})$ [23].

Lemma 1. [25] Let a gain matrix $K_x(\boldsymbol{x}) : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ have the following structure

$$K_x(\boldsymbol{x}) = \begin{bmatrix} k_{x_1}(x_1) & 0 & \cdots & 0 \\ 0 & k_{x_2}(x_2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & k_{x_n}(x_n) \end{bmatrix}. \qquad (3)$$

Assume that there exist constants k_{l_i} and k_{u_i} where $k_{u_i} > k_{l_i} > 0$ such that $k_{u_i} \geq k_{x_i}(x_i) \geq k_{l_i}$ for all $x_i \in \mathbb{R}$ and $i = 1,\cdots,n$, then

$$\frac{1}{2}k_{u_i}\left|x_i\right|^2 \geq \int_0^{x_i}\xi_i k_{x_i}(\xi_i)\,d\xi_i \geq \frac{1}{2}k_{l_i}\left|x_i\right|^2, \qquad (4)$$

Definition:1 [26] $F\left(m,\varepsilon,x\right)$with $1 \geq m > 0$, $\varepsilon > 0$ and $x \in \mathrm{R}^n$ denote the set of all continuously differentiable increasing functions

$$sat\left(x\right) = \begin{vmatrix} sat\left(x_1\right) \\ sat\left(x_2\right) \\ \vdots \\ sat\left(x_n\right) \end{vmatrix}$$

Such that

- $|x| \geq sat\left(x\right) > m\left|x\right|$, for all $x \in \mathbb{R} : |x| < \varepsilon$,
- $\varepsilon \geq \left|sat\left(x\right)\right| \geq m\varepsilon$ for all $x \in \mathbb{R} : |x| \geq \varepsilon$,
- $1 \geq \frac{d(sat(x))}{dx} > 0$ for all $x \in \mathbb{R}$,

Assumption 1. There exist positive constants k_{pl_i} and k_{pu_i} such that Lemma 1 can be applied. That is:

$$\frac{1}{2}\tilde{\boldsymbol{q}}^T K_{pu}\tilde{\boldsymbol{q}} \geq \int_0^{\tilde{\boldsymbol{q}}}\boldsymbol{\xi}K_p(\boldsymbol{\xi})\,d\boldsymbol{\xi} \geq \frac{1}{2}\tilde{\boldsymbol{q}}^T K_{pl}\tilde{\boldsymbol{q}} \qquad (5)$$

where K_{pu}, K_{pl} are $n\times n$ constant positive definite diagonal matrices whose entries are k_{pu_i}, k_{pl_i} respectively, with $i = 1, 2, ...n$.

III. PID CONTROL WITH NONLINEAR GAINS

The PID controller is a well known set point control strategy for manipulators which ensures asymptotic stability for fixed symmetric positive definite gain matrices. In order to improve the performance of the closed loop system, it may be necessary to have variable gains. [17] In this section we introduce a new PID controller whose main feature is that stability holds even though the parameters depend on the robot state. A some case of generalization of the classical linear PID controller can be obtained by allowing to have nonlinear proportional $K_p(\tilde{\boldsymbol{q}})$ and derivative $K_v(\tilde{\boldsymbol{q}})$ gain matrices and constant integral gain matrix K_i as function matrices of the robot configuration. This leads to the following proposed control law

$$\boldsymbol{\tau} = K_p(\tilde{\boldsymbol{q}})\tilde{\boldsymbol{q}} - K_v(\tilde{\boldsymbol{q}})\dot{\boldsymbol{q}} + K_i\int_0^t\left(\alpha sat(\tilde{\boldsymbol{q}}(\sigma)) + \dot{\tilde{\boldsymbol{q}}}(\sigma)\right)d\sigma \quad (6)$$

where $K_p(\tilde{\boldsymbol{q}})$, $K_v(\tilde{\boldsymbol{q}})$ and K_i are positive definite diagonal $n\times n$ matrices, whose entries are denoted by $k_{p_i}(\tilde{q}_i)$, $k_{v_i}(\tilde{q}_i)$ and k_{i_i} respectively, and $\tilde{\boldsymbol{q}} = \boldsymbol{q}_d - \boldsymbol{q}$ denotes the position error vector, with $\alpha > 0$.

The closed- loop system is obtained substituting the control law (6) into the robot dynamics (1), see figure 1 . This can be written as

$$\frac{d}{dt}\begin{bmatrix} \tilde{\boldsymbol{q}} \\ \dot{\boldsymbol{q}} \\ \dot{\boldsymbol{\omega}} \end{bmatrix} = \begin{bmatrix} -\dot{\boldsymbol{q}} \\ M^{-1}\left[K_p(\tilde{\boldsymbol{q}})\tilde{\boldsymbol{q}} - K_v(\tilde{\boldsymbol{q}})\dot{\boldsymbol{q}} - C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \boldsymbol{g}(\boldsymbol{q}) \right. \\ \left. + K_i\boldsymbol{\omega} + \boldsymbol{g}(\boldsymbol{q}_d)\right] \\ \alpha sat(\tilde{\boldsymbol{q}}) - \dot{\boldsymbol{q}} \end{bmatrix} \qquad (7)$$

where $\boldsymbol{\omega}$ is defined as:

$$\boldsymbol{\omega}(t) = \int_0^t\left[\alpha sat(\tilde{\boldsymbol{q}}(\sigma)) + \dot{\tilde{\boldsymbol{q}}}(\sigma)\right]d\sigma - K_i^{-1}\boldsymbol{g}(\boldsymbol{q}_d)$$

and we have that (7) becomes an autonomous nonlinear differential equation whose origin

$$\left[\,\tilde{\boldsymbol{q}}^T \quad \dot{\boldsymbol{q}}^T \quad \boldsymbol{\omega}^T\,\right]^T = \boldsymbol{0} \in \mathbb{R}^{3n} \qquad (8)$$

is an equilibrium point.

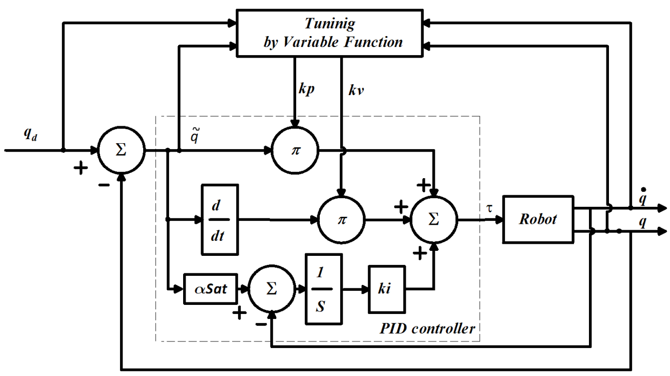

Fig. 1. Block diagram: regulator in closed-loop.

IV. GLOBAL ASYMPTOTIC STABILITY ANALYSIS

In this section we show that the stability also holds for a class of nonconstant state-depending proportional and derivative gain matrices. More specifically, consider the control law (6) corresponding to a PID control scheme with nonlinear gain matrices. The stability analysis is inspired in [27].

A. Lyapunov function candidate

In order to study stability of equilibrium point (8) we propose the following Lyapunov function candidate:

$$\begin{aligned} V(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{\omega}) &= \int_{\mathbf{0}}^{\tilde{\boldsymbol{q}}} \boldsymbol{\xi}^T K_p(\boldsymbol{\xi})\, d\boldsymbol{\xi} - \mathcal{U}(\boldsymbol{q}_d) + \mathcal{U}(\boldsymbol{q}) \\ &\quad + \boldsymbol{g}(\boldsymbol{q}_d)^T\tilde{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} \\ &\quad -\alpha sat(\tilde{\boldsymbol{q}})^T M(\boldsymbol{q})\dot{\boldsymbol{q}} \\ &\quad +\frac{1}{2}\boldsymbol{\omega}^T K_i \boldsymbol{\omega} \end{aligned} \tag{9}$$

Note that under Assumption 1 the function in (9) can be lower bounded as:

$$\begin{aligned} V(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{\omega}) \geq\ & V_L(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{\omega}) = \frac{1}{2}\tilde{\boldsymbol{q}}^T K_{pl}\tilde{\boldsymbol{q}} - \mathcal{U}(\boldsymbol{q}_d) + \mathcal{U}(\boldsymbol{q}) \\ & + \frac{1}{2}\left[\dot{\boldsymbol{q}} - \alpha sat(\tilde{\boldsymbol{q}})\right]^T M(\boldsymbol{q})\left[\dot{\boldsymbol{q}} - \alpha sat(\tilde{\boldsymbol{q}})\right] \\ & -\frac{\alpha^2}{2} sat(\tilde{\boldsymbol{q}})^T M(\boldsymbol{q}) sat(\tilde{\boldsymbol{q}}) \\ & +\frac{1}{2}\boldsymbol{\omega}^T K_i\boldsymbol{\omega} + \boldsymbol{g}(\boldsymbol{q}_d)^T\tilde{\boldsymbol{q}} \end{aligned} \tag{10}$$

Now, we will give sufficient conditions to make $V_L(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{\omega})$ be a positive definite function.

Term $\frac{1}{2}\left[\dot{\boldsymbol{q}} - \alpha sat(\tilde{\boldsymbol{q}})\right]^T M(\boldsymbol{q})\left[\dot{\boldsymbol{q}} - \alpha sat(\tilde{\boldsymbol{q}})\right]$ is a positive definite matrix. Due to K_i is assumed > 0 hence $\frac{1}{2}\boldsymbol{\omega}^T K_i\boldsymbol{\omega}$ is positive definite.

Let us define $U_T(\tilde{\boldsymbol{q}},\boldsymbol{q}_d)$ as the virtual total potential energy [28] consisting of the sum of the robot gravitational potential energy $U_a(\boldsymbol{q}_d,\tilde{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q})^T\tilde{\boldsymbol{q}}$ plus the artificial potential energy $U(\boldsymbol{q}_d,\tilde{\boldsymbol{q}})$ induced by the controller, then

$$U_T(\tilde{\boldsymbol{q}},\boldsymbol{q}_d) = \tfrac{1}{2}\tilde{\boldsymbol{q}}^T K_{pl}\tilde{\boldsymbol{q}} - \mathcal{U}(\boldsymbol{q}_d) + \mathcal{U}(\boldsymbol{q}) + \boldsymbol{g}(\boldsymbol{q}_d)^T\tilde{\boldsymbol{q}}$$

where $U_T(\tilde{\boldsymbol{q}},\boldsymbol{q}_d)$ has the following property [28]

$$U_T(\tilde{\boldsymbol{q}},\boldsymbol{q}_d) - U_T(\boldsymbol{q}_d,0) \geq \beta\left\|sat(\tilde{\boldsymbol{q}})\right\|^2 \forall\tilde{\boldsymbol{q}} \in \mathbb{R}^n \tag{11}$$

where $\beta \geq \frac{1}{2}\left[\lambda_m\{K_p\} - k_g\right]$, and next term can be lower bounded as:

$$-\frac{\alpha^2}{2} sat(\tilde{\boldsymbol{q}})^T M(\boldsymbol{q}) sat(\tilde{\boldsymbol{q}}) \geq -\frac{\alpha^2}{2}\lambda_M\{M(\boldsymbol{q})\}\left\|sat(\tilde{\boldsymbol{q}})\right\|^2 \tag{12}$$

from (11) and (12) is gotten

$$\beta\left\|sat(\tilde{\boldsymbol{q}})\right\|^2 - \frac{\alpha^2}{2}\lambda_M\{M(\boldsymbol{q})\}\left\|sat(\tilde{\boldsymbol{q}})\right\|^2 \geq 0$$

$$\beta - \frac{\alpha^2}{2}\lambda_M\{M(\boldsymbol{q})\} \geq 0$$

$$\sqrt{\frac{2\beta}{\lambda_M\{M(\boldsymbol{q})\}}} \geq \alpha \tag{13}$$

thus

$$V(\tilde{q},\dot{q},w) > 0$$

In sum, the Lyapunov function candidate (9) is a globally positive definite function under the condition (13) before mentioned.

B. Time derivative of the Lyapunov function candidate

The time derivative of the Lyapunov function candidate (9) along the trajectories of the closed loop equation (7) is

$$\begin{aligned} \dot{V}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{\omega}) &= \tilde{\boldsymbol{q}}^T K_p(\tilde{\boldsymbol{q}})\dot{\tilde{\boldsymbol{q}}} + \boldsymbol{g}(\boldsymbol{q})^T\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}_d)^T\dot{\tilde{\boldsymbol{q}}} \\ &\quad +\frac{1}{2}\dot{\boldsymbol{q}}^T\dot{M}(\boldsymbol{q})\dot{\boldsymbol{q}} - \alpha s\dot{a}t(\tilde{\boldsymbol{q}})^T M(\boldsymbol{q})\dot{\boldsymbol{q}} \\ &\quad +\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\ddot{\boldsymbol{q}} - \alpha sat(\tilde{\boldsymbol{q}})^T\dot{M}(\boldsymbol{q})\dot{\boldsymbol{q}} \\ &\quad -\alpha sat(\tilde{\boldsymbol{q}})^T M(\boldsymbol{q})\ddot{q} + \boldsymbol{w}^T K_i\dot{\boldsymbol{w}} \end{aligned} \tag{14}$$

where we have used the Leibnitz's rule for differentiation of integrals and Property 1.

So far, substituting the expanded value of $\ddot{\boldsymbol{q}}$ in (14), we have the following:

$$\begin{aligned} \dot{V}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{w}) = &\ \tilde{\boldsymbol{q}}^T K_p(\tilde{\boldsymbol{q}})\dot{\tilde{\boldsymbol{q}}} + \boldsymbol{g}(\boldsymbol{q})^T\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}_d)^T\dot{\tilde{\boldsymbol{q}}} \\ & +\tfrac{1}{2}\dot{\boldsymbol{q}}^T\dot{M}(\boldsymbol{q})\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T[K_p(\tilde{\boldsymbol{q}})\tilde{\boldsymbol{q}} - \boldsymbol{g}(\boldsymbol{q}) \\ & -K_v(\tilde{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}_d) + K_i\boldsymbol{w} - C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}] \\ & -\alpha s\dot{a}t(\tilde{\boldsymbol{q}})^T M(\boldsymbol{q})\dot{\boldsymbol{q}} \\ & -\alpha sat(\tilde{\boldsymbol{q}})^T\dot{M}(\boldsymbol{q})\dot{\boldsymbol{q}} + w^T K_i\dot{\boldsymbol{w}} \\ & -\alpha sat(\tilde{\boldsymbol{q}})^T[K_p(\tilde{\boldsymbol{q}})\tilde{\boldsymbol{q}} - K_v(\tilde{\boldsymbol{q}})\dot{\boldsymbol{q}} \\ & +\boldsymbol{g}(\boldsymbol{q}_d) + K_i\boldsymbol{w} - C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - g(\boldsymbol{q})] \\ & +\boldsymbol{w}^T K_i\dot{\boldsymbol{w}} \end{aligned} \tag{15}$$

a simplified result is

$$\begin{aligned} \dot{V}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{w}) &= -\dot{\boldsymbol{q}}^T K_v(\tilde{\boldsymbol{q}})\dot{\boldsymbol{q}} \\ &\quad +\alpha sat(\tilde{\boldsymbol{q}})^T K_v(\tilde{\boldsymbol{q}})\dot{\boldsymbol{q}} \\ &\quad -\alpha s\dot{a}t(\tilde{\boldsymbol{q}})^T M(\boldsymbol{q})\dot{\boldsymbol{q}} \\ &\quad -\alpha sat(\tilde{\boldsymbol{q}})^T C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} \\ &\quad -\alpha sat(\tilde{\boldsymbol{q}})^T K_p(\tilde{\boldsymbol{q}})\tilde{\boldsymbol{q}} \\ &\quad -\alpha sat(\tilde{\boldsymbol{q}})^T \boldsymbol{g}(\boldsymbol{q}_d) \\ &\quad +\alpha sat(\tilde{\boldsymbol{q}})^T \boldsymbol{g}(\boldsymbol{q}) \end{aligned} \tag{16}$$

notice that

$$
\begin{array}{l}
-\dot{\boldsymbol{q}}^T Kv\left(\tilde{\boldsymbol{q}}\right)\dot{\boldsymbol{q}} + \alpha sat\left(\tilde{\boldsymbol{q}}\right)^T Kv\left(\tilde{\boldsymbol{q}}\right)\dot{\boldsymbol{q}} = \\
-\frac{1}{2}\dot{\boldsymbol{q}}^T K_v\left(\tilde{\boldsymbol{q}}\right)\dot{\boldsymbol{q}} + \alpha sat\left(\tilde{\boldsymbol{q}}\right)^T K_v\left(\tilde{\boldsymbol{q}}\right)\dot{\boldsymbol{q}} \\
-\frac{1}{2}\dot{\boldsymbol{q}}^T K_v\left(\tilde{\boldsymbol{q}}\right)\dot{\boldsymbol{q}}
\end{array} \tag{17}
$$

and

$$
\begin{array}{l}
-\frac{1}{2}\dot{\boldsymbol{q}}^T K_v\left(\tilde{\boldsymbol{q}}\right)\dot{\boldsymbol{q}} + \alpha sat\left(\tilde{\boldsymbol{q}}\right)^T K_v\left(\tilde{\boldsymbol{q}}\right)\dot{\boldsymbol{q}} \\
= -\frac{1}{2}\left[\dot{\boldsymbol{q}} - \alpha sat\left(\tilde{\boldsymbol{q}}\right)\right]^T K_v\left(\tilde{\boldsymbol{q}}\right)\left[\dot{\boldsymbol{q}} - \alpha sat\left(\tilde{\boldsymbol{q}}\right)\right] \\
+\frac{\alpha^2}{2} sat\left(\tilde{\boldsymbol{q}}\right)^T K_v\left(\tilde{\boldsymbol{q}}\right) sat\left(\tilde{\boldsymbol{q}}\right)
\end{array}
$$

then it's possible to write (16) as

$$
\begin{aligned}
\dot{V}\left(\tilde{\boldsymbol{q}}, \dot{\boldsymbol{q}}, w\right) \;=\;& -\alpha s\dot{a}t\left(\tilde{\boldsymbol{q}}\right)^T M\left(\boldsymbol{q}\right)\dot{\boldsymbol{q}} \\
& -\alpha s\dot{a}t\left(\tilde{\boldsymbol{q}}\right)^T C^T\left(\dot{\boldsymbol{q}}, \boldsymbol{q}\right)\dot{\boldsymbol{q}} \\
& -\alpha sat\left(\tilde{\boldsymbol{q}}\right)^T\left[\boldsymbol{g}(\boldsymbol{q}_d) - \boldsymbol{g}(\boldsymbol{q})\right] \\
& -\alpha sat\left(\tilde{\boldsymbol{q}}\right)^T K_p\left(\tilde{\boldsymbol{q}}\right)\tilde{\boldsymbol{q}} - \frac{1}{2}\dot{\boldsymbol{q}}^T K_v\left(\tilde{\boldsymbol{q}}\right)\dot{\boldsymbol{q}} \\
& -\frac{1}{2}\left[\dot{\boldsymbol{q}} - \alpha sat\left(\tilde{\boldsymbol{q}}\right)\right]^T K_v\left(\tilde{\boldsymbol{q}}\right)\left[\dot{\boldsymbol{q}} - \alpha sat\left(\tilde{\boldsymbol{q}}\right)\right] \\
& +\frac{\alpha^2}{2} sat\left(\tilde{\boldsymbol{q}}\right)^T K_v\left(\tilde{\boldsymbol{q}}\right) sat\left(\tilde{\boldsymbol{q}}\right)
\end{aligned} \tag{18}
$$

Now we provide upper bounds on the following terms

$$-\alpha s\dot{a}t\left(\tilde{\boldsymbol{q}}\right)^T M\left(\boldsymbol{q}\right)\dot{\boldsymbol{q}} \leq \alpha\lambda_M\left\{M\left(\boldsymbol{q}\right)\right\}\left\|\dot{\boldsymbol{q}}\right\|^2$$

$$-\alpha s\dot{a}t\left(\tilde{\boldsymbol{q}}\right)^T C^T\left(\dot{\boldsymbol{q}}, \boldsymbol{q}\right)\dot{\boldsymbol{q}} \leq \alpha K_c\sqrt{n}\zeta\left\|\dot{\boldsymbol{q}}\right\|^2$$

$$-\frac{1}{2}\dot{\boldsymbol{q}}^T K_v\left(\tilde{\boldsymbol{q}}\right)\dot{\boldsymbol{q}} \leq -\frac{1}{2}\lambda_M\left\{K_v\left(\tilde{\boldsymbol{q}}\right)\right\}\left\|\dot{\boldsymbol{q}}\right\|^2$$

an upper bound for gravity is given as

$$
\begin{array}{l}
-\alpha sat\left(\tilde{\boldsymbol{q}}\right)^T\left[g(\boldsymbol{q}_d) - g(\boldsymbol{q})\right] \leq \left\|\alpha sat\left(\tilde{\boldsymbol{q}}\right)^T\left[g(\boldsymbol{q}_d) - g(\boldsymbol{q})\right]\right\| \\
\left\|\alpha sat\left(\tilde{\boldsymbol{q}}\right)^T\left[g(\boldsymbol{q}_d) - g(\boldsymbol{q})\right]\right\| \leq \alpha\left\|sat\left(\tilde{\boldsymbol{q}}\right)\right\|\left\|g(\boldsymbol{q}_d) + g(\boldsymbol{q})\right\| \\
\alpha\left\|sat\left(\tilde{\boldsymbol{q}}\right)\right\|\left\|g(\boldsymbol{q}_d) + g(\boldsymbol{q})\right\| \leq \alpha\left\|sat\left(\tilde{\boldsymbol{q}}\right)\right\| K_{h2}\left\|sat\left(\tilde{\boldsymbol{q}}\right)\right\| \\
= \alpha K_{h2}\left\|sat\left(\tilde{\boldsymbol{q}}\right)\right\|^2
\end{array}
$$

where K_{h2} had been defined in [21]. A bound for this next term is

$$-\alpha sat\left(\tilde{\boldsymbol{q}}\right)K_p\left(\tilde{\boldsymbol{q}}\right)\boldsymbol{q} \leq -\alpha sat\left(\tilde{\boldsymbol{q}}\right)K_p\left(\tilde{\boldsymbol{q}}\right) sat\left(\tilde{\boldsymbol{q}}\right)$$

then we can apply Rayleigh-Ritz theorem as follow

$$-\alpha sat\left(\tilde{\boldsymbol{q}}\right)K_p\left(\tilde{\boldsymbol{q}}\right)\boldsymbol{q} \leq -\alpha\lambda_m\left\{K_p\left(\tilde{\boldsymbol{q}}\right)\right\}\left\|sat\left(\tilde{\boldsymbol{q}}\right)\right\|^2$$

For last term

$$\frac{1}{2}\alpha^2 sat\left(\tilde{\boldsymbol{q}}\right)^T K_v\left(\tilde{\boldsymbol{q}}\right) sat\left(\tilde{\boldsymbol{q}}\right)$$

$$\leq \frac{1}{2}\alpha^2\lambda_m\left\{K_v\left(\tilde{\boldsymbol{q}}\right)\right\}\left\|sat\left(\tilde{\boldsymbol{q}}\right)\right\|^2 \frac{1}{2}\alpha^2\lambda_m\left\{K_v\left(\tilde{\boldsymbol{q}}\right)\right\}\left\|sat\left(\tilde{\boldsymbol{q}}\right)\right\|^2$$

$$\leq \frac{1}{2}\alpha^2\lambda_m\left\{K_v\left(\tilde{\boldsymbol{q}}\right)\right\}\left\|\tilde{\boldsymbol{q}}\right\|^2$$

This allow us establish that, if

$$\frac{\frac{1}{2}\lambda_m\left\{K_v\left(\tilde{\boldsymbol{q}}\right)\right\}}{\left(\lambda_M\left\{M\left(\boldsymbol{q}\right)\right\} + K_c\sqrt{n}\zeta\right)} > \alpha \tag{19}$$

and

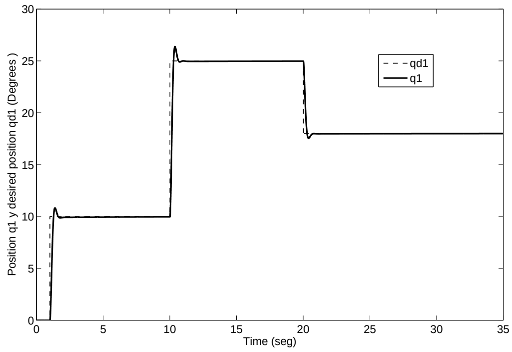

Fig. 2. Position of join q_1 and desired position q_{d1} .

$$\frac{2\left[\lambda_m\left\{K_p\left(\tilde{\boldsymbol{q}}\right)\right\} - K_{h2}\right]}{\lambda_M\left\{K_v\left(\tilde{\boldsymbol{q}}\right)\right\}} > \alpha \tag{20}$$

then $\dot{V}\left(\tilde{\boldsymbol{q}}, \dot{\boldsymbol{q}}, w\right)$ is a globally negative semidefinite function.

Using the fact that the Lyapunov function candidate (9) is radially unbounded globally positive definite and its time derivative is a globally negative semidefinite function, we conclude that the equilibrium of the closed-loop system (7) is stable.

Finally, by invoking the Lasalle's invariance principle, we conclude that the equilibrium of the closed-loop system is globally asymptotic stable.

Note. This system has been analyzed no for following of trajectories. It will be used only for regulation, following of one random fix point to other one.

V. SIMULATION RESULTS

Computer simulations have been carried out to show the stability and performance of the PID regulator. The manipulator used for this simulation is a robot arm with two revolute joins (planar elbow manipulator). The robot and their numerical values are the same as used in [17].

In the next figures 2 and 3 the behavior of this regulator is showed, where its clear to see than its good.

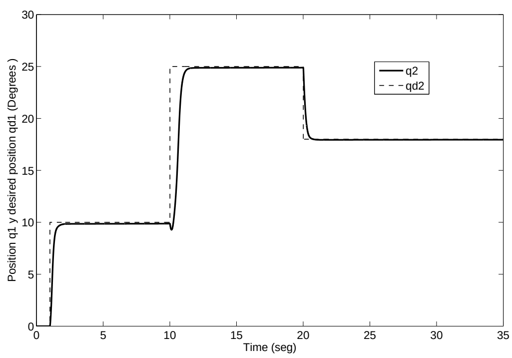

Fig. 3. Position of join q_2 and desired position q_{d2} .

For the join closer to the base (sub subscript 1 were added in order to refer at this join), the gains used were the following equations :

$$K_{p1} = 293 - 200\tilde{q_1}$$

$$K_{v1} = 36 - 5\dot{q_1}$$

$$K_{i1} = 35$$

For the join farther to the base (sub subscript 2 were added in order to refer at this join), the gains used were the following equations :

$$K_{p1} = 90 - 320\tilde{q_2}$$

$$K_{v1} = 15 - 8\dot{q_2}$$

$$K_{i1} = 35$$

the values of α and ζ and n were 1, 40 and 2 respectively.

VI. CONCLUSIONS

In this paper we have proposed a new nonlinear PID regulator, a little bit more generalized in sense of it admit variable gains for proportional and derivative terms, for robots manipulators. Additional, here had been mathematically demonstrated that this system in closed-loop, with the founded conditions, its equilibrium is globally asymptotic stable, following to the Lyapunov principles.

Now we are working in the feasibility for real time implementation using this control strategy. It will be implemented in a robot from Tecnlogico de la Laguna. It's a similar to CICESE's robot of two degrades of freedom.

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